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ENERGETIC EQUIVALENCE OF POLAR AND LATERAL QUANTITIES ON PERMANENT MAGNET BOUNDARIES

SUMMARY *It is proved that for permanent magnets of cylindrical shape set in free space two virtual boundary quantities: surface density of magnetic charge that is attributed to a polar surface, and magnetizing current sheet imagined on a lateral surface produce both the identical magnetic energy. The exact formulae to compute the magnetic energy are given. Results of test calculations performed by the use of author's test software are presented.*

Notation

$\mathcal{R}, \mathcal{S}, \mathcal{E}$ – symbols of regions, surfaces and edges, respectively,

r, φ, z – cylindrical co-ordinates,

r_m – radius of a cylindrical magnet,

l_m – length of a cylindrical magnet,

B – magnetic flux density,

H – magnetic field strength,

M – magnetization,

J – magnetic polarization,

W_m – magnetic energy,

σ_m – surface monopole density of a "magnetic charge",

$K_M = e_\varphi K_M$ – magnetizing current sheet, A/m,

ψ_m – magnetic scalar potential,

A – magnetic vector potential,

$\mu_0 = 4\pi \cdot 10^{-7}$ H/m – magnetic constant,

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$J_m(ru)$ – Bessel function of the real argument ru and of the rang m (non-negative integer),

$\int_0^\infty J_m(r_i u) J_n(r_j u) e^{-z u} u^\delta du$ – integral of Lipschitz-Hankel type.

Complete elliptic integrals of argument k of:

the first kind: $K(k) = \int_0^{\pi/2} \frac{d\phi}{\sqrt{1-k^2 \sin^2 \phi}}$; $K_0(k) = \frac{2}{\pi} K(k)$

the second kind: $E(k) = \int_0^{\pi/2} \sqrt{1-k^2 \sin^2 \phi} d\phi$; $E_0(k) = \frac{2}{\pi} E(k)$.

1. INTRODUCTION – TWO BOUNDARY QUANTITIES

Consider a cylindrical permanent magnet that is set in free space, shown in Fig.1. Let the magnet be flat ($l_m < r_m$) and sufficiently small to suppose be uniformly magnetized in axial direction, it means that two magnet quantities inside the whole magnet body $\mathfrak{R}^{(m)}$ are steadily distributed, i.e. the magnetization $\mathbf{M}(r, \varphi, z) = k \mathbf{M}_z = k \mathbf{M} = \text{const}$, as well as the magnetic polarization $\mathbf{J}(r, \varphi, z) = k \mathbf{J}_z = k \mathbf{J} = \text{const}$. The values of these quantities are not given *a priori*, but they shall be determined from the magnet size and its magnetic curve – see [9]. It is not to conclude from the above that the magnet quantities $\mathbf{B}$ and $\mathbf{H}$ would be constant inside $\mathfrak{R}^{(m)}$, indeed.

Two equivalent boundary models of the permanent magnet will be used in the form that was defined and published by the author [1], [2], [3], [9].

To perform the first model with analogy between magneto- and electrostatic approach, let us introduce for the magnet a fictitious polar quantity attributed to the magnet polar surfaces $\mathfrak{G}^{(N)}$ and $\mathfrak{G}^{(S)}$ and termed *surface monopole density of the magnetic charge*, symbol σ_m , unit tesla¹⁾. When the magnetic polarization $\mathbf{J}$ is directed according z-axis i.e. from S to N, we have: $\sigma_{mN} = \sigma_m = k \cdot \mathbf{J} = J$, for N-pole, and $\sigma_{mS} = -\sigma_m = -k \cdot \mathbf{J} = -J$, for S-pole.

In conformity with an interpretation of σ_m as a quantity of mono-pole density type the magnetic scalar potential depending on σ_{mN} can be introduced as having a form of the integral of Lipschitz-Hankel type – see [6], [9]:

¹⁾ Craik [6] employs here the quantity *pole density*, designated σ without an index and expressed in A/m, but equal to σ_m/μ_0 in our symbols; other authors, e.g. Abele [12] prefer our approach.

$$\psi_m(r, z) = \frac{r_m \sigma_{mN}}{2\mu_0} \int_0^\infty J_1(r_m, u) J_0(r, u) e^{-|z-l|u} u^{-1} du \quad (1)$$

where $l = l_m/2$, and the beginning of z -axis is chosen at the magnet centre. The magnetic potential given by (1) is unique; it vanishes for $|z| \rightarrow \infty$.

Formula (1) can be simply generalized for both polar surfaces expressing the magnetic scalar potential created by the whole permanent magnet to read:

$$\psi_m(r, z) = \frac{r_m \sigma_m}{2\mu_0} \int_0^\infty J_1(r_m, u) J_0(r, u) \left(e^{-|z-l|u} - e^{-|z+l|u} \right) u^{-1} du \quad (1a)$$

In the second model of the permanent magnet some vector boundary quantity could be imagined upon a lateral surface $\mathfrak{G}^{(L)}$; it is commonly termed *magnetizing current sheet*²⁾, designated by K_M and defined by the following way:

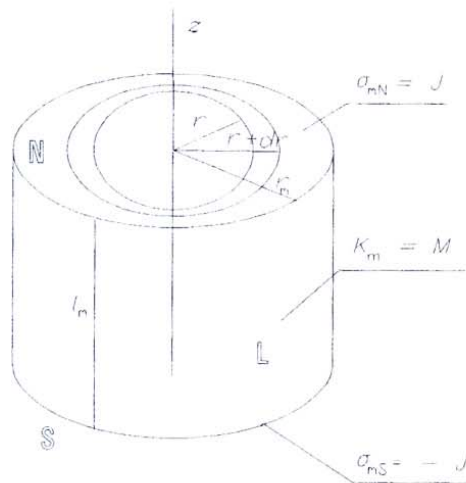


Fig.1. Virtual boundary quantities on the pole and lateral surface of a permanent magnet.

²⁾ Magnetizing current sheet is also termed linear density of magnetizing current.

$\mathbf{K}_M = \vec{e}_\varphi K_M = \vec{e}_r \times \mathbf{M}_z$, thus $K_M = M_z = M$, unit ampere per metre. K_M shall be interpreted as a quantity creating double-layer potential. This boundary quantity excites the rotational field $\mathbf{B}$ in the magnet body and in external space.

For the second model, the magnetic vector potential depending on K_M may be introduced. Choosing optionally $z = 0$ at the centre of S-pole surface the following integral expression for $A_\varphi = A$ holds:

$$A(r, z) = \frac{\mu_0 r_m K_M}{2} \int_0^\infty J_1(r_m u) J_1(r u) e^{-(l_m - z)u} u^{-1} du \quad (2)$$

that can be easily drawn up from the relevant formula of the magnetic vector potential created by an electric current carried by a circular loop – see [9].

Magnetic vector potential given by formula (2) is unique and vanishing for $|z| \rightarrow \infty$. The Lipschitz-Hankel integral occurring in (2) differs from that in (1a) by the order of the second Bessel function.

2. MAGNETIC ENERGY DUE TO σ_m

The polar densities distributed uniformly on the polar surfaces are considered as unique sources of the magnetic energy in $\mathfrak{R}^{(3)}$, thus, the magnetic energy can be evaluated by integration of the magnetic field upon the both N and S polar surfaces.

The magnetic energy W_m created by the inherent magnetization of the magnet alone and stocked in the whole $\mathfrak{R}^{(3)}$ should be determined as a sum of four terms:

$$W_m = W_N + W_S + W_{NS} + W_{NS} \quad (3)$$

where:

W_N – self energy of the N-pole surface,

W_S – self energy of the S-pole surface,

W_{NS} – mutual energy created by the σ_{mS} acting on the N-pole surface,

W_{SN} – mutual energy created by the σ_{mN} acting on the S-pole surface³⁾.

³⁾ Some authors prefer to employ the concept W'_M identical to $W_{NS} + W_{SN}$ in our designation.

To summarise, we have readily $W_N = W_S$ and similarly $W_{SN} = W_{NS}$, hence:

$$W_m = 2(W_N + W_{SN}) \quad (4)$$

To obtain W_N , let us firstly express dW_N related to the small ring on the N-pole surface – see Fig.1. Putting optionally $z = 0$ at the N-surface centre we have:

$$dW_N = \frac{1}{2} \psi_m(r, 0) \sigma_{mN} \cdot 2\pi r dr \quad (5)$$

where $\psi_m(r, 0)$ is the magnetic scalar potential excited by σ_{mN} in any point on the N-pole surface – compare [9] – and it can be expressed in the form of integral of Lipschitz-Hankel type to read:

$$W_N = \frac{\pi r_m^2 \sigma_m^2}{2\mu_0} \int_0^\infty J_1(r_m u) \left[\int_0^{r_m} J_0(r u) r dr \right] u^{-1} du \quad (6)$$

In the considered case, we have $r < r_m$ and moreover $\sigma_{mN}^2 = \sigma_m^2$, hence, we obtain the formula:

$$W_N = \frac{\pi r_m^2 \sigma_m^2}{2\mu_0} \int_0^\infty [J_1(r_m u)]^2 u^{-2} du \quad (7)$$

Similar procedure is to apply to express W_{SN} . Choosing for this case $z = 0$ at the centre of the S-pole surface we have now $z = l_m$ and $\sigma_{mN}\sigma_{mS} = -\sigma_m^2$; we get:

$$W_{SN} = -\frac{\pi r_m^2 \sigma_m^2}{2\mu_0} \int_0^\infty [J_1(r_m u)]^2 e^{-l_m u} u^{-2} du \quad (8)$$

In conclusion, the magnetic energy defined by (4) can be written with the use of the integral of Lipschitz-Hankel type in the following form:

$$W_m = \frac{\pi r_m^2 \sigma_m^2}{\mu_0} \int_0^\infty [J_1(r_m u)]^2 (1 - e^{-l_m u}) u^{-2} du \quad (9)$$

Passing to classical quantities commonly used to describe the magnetic field inside the magnet body $\mathfrak{R}^{(m)}$ we can substitute $\sigma_m^2/\mu_0 = JM$ to reach:

$$W_m = \pi r_m^2 JM \int_0^\infty [J_1(r_m u)]^2 (1 - e^{-l_m u}) u^{-2} du \quad (9a)$$

Expression (9a) determines exactly the magnetic energy stored in $\mathfrak{R}^{(3)}$, when the permanent magnet of circular shape is uniformly magnetized, set in free space, and not influenced by external fields.

3. MAGNETIC ENERGY DUE TO K_M

One can show that for the second model based on the fictitious lateral quantity interpreted as the magnetizing current sheet K_M the similar procedure is to apply. Starting from the expression valid for one K_M -layer:

$$dW_m = \frac{1}{2} A(r_m, z) K_M \cdot 2\pi r_m dz \quad (10)$$

we can employ magnetic vector potential given by formula (2) for $r = r_m$, hence, for double layer resulting in multiplying the expression by 2 we arrive to:

$$W_m = \mu_0 \pi r_m^2 K_M^2 \int_0^\infty [J_1(r_m u)]^2 \left[\int_0^{l_m} e^{-(l_m - z)u} dz \right] u^{-1} du \quad (11)$$

thus, after integrating the relevant part with respect to z we obtain:

$$W_m = \mu_0 \pi r_m^2 K_M^2 \int_0^\infty [J_1(r_m u)]^2 (1 - e^{-l_m u}) u^{-2} du \quad (12)$$

It is to conclude that for the uniformly magnetized permanent magnet of cylindrical shape which is set in free space and is not influenced by external fields, we can substitute $\mu_0 K_M^2 = JM$ to prove full equivalence between W_m given by (9) or (12), respectively, in the common form (9a).

4. APPROXIMATION AND PHYSICAL INTERPRETATION

In order to become the formula (9a) in a form admitting the use of the elliptic integrals we can accept the approximation:

$$e^{-l_m u} \approx 1 - l_m u + \frac{1}{2}(l_m u)^2 \mp \dots \quad (13)$$

It is convergent in the case of flat magnets for which $r_m > l_m$. Thus, we would like to substitute into (9a):

$$1 - e^{-l_m u} \approx l_m u - \frac{1}{2}(l_m u)^2 \pm \dots \quad (13a)$$

The permutation of signs occurs in series (13). Thus, when we substitute into (9a): $1 - e^{-l_m u} \approx l_m u$ only, we are able to interpret the formula as an upper band of the exact value of the magnetic energy, and putting there two first terms written in (13a) – as lower band of it. Therefore, let us to rewrite formula (9a) as follows:

$$W_m \approx \frac{\pi r_m^2 l_m \sigma_m^2}{2\mu_0} \left\{ 2 \int_0^\infty [J_1(r_m u)]^2 u^{-1} du - l_m \int_0^\infty [J_1(r_m u)]^2 du \right\} \quad (9b)$$

The first integral is equal to $\frac{1}{2}$ – see [10, Int. no 4.416.2] – thus, the first term in the braces is simply equal to 1. Substituting $\sigma_m^2 = J^2$, neglecting the second term and symbolising the magnet volume by V_m we obtain, in consequence, the upper band of the magnetic energy as:

$$W'_m = \frac{\pi r_m^2 l_m J^2}{2\mu_0} = \frac{V_m J^2}{2\mu_0} = \frac{V_m}{2} J M \quad (14)$$

that was shown by Craik in [6] as the approximate value of magnetic energy. We prefer to interpret it as the upper band of the magnetic energy stored in the

whole $\Re^3$ and created by the yokeless permanent magnet of a cylindrical shape that is uniformly magnetized in its axial direction:

$$W_m^{(+)} = \frac{V_m}{2} J M \quad (15)$$

The second integral in the braces in (9b) can be expressed by means of the elliptic integrals: of the first kind $K_0(k) = \frac{2}{\pi} K(k)$ and similarly of the second kind $E_0(k) = \frac{2}{\pi} E(k)$ with the argument $k^2 = \frac{4r_m^2}{4r_m^2 + l_m^2}$ – see [9] – hence, an amendment W_m'' will reach:

$$W_m'' = W_m' \frac{l_m}{r_m k} \left[\left(1 - \frac{1}{2} k^2 \right) K_0(k) - E_0(k) \right] \quad (16)$$

that leads to the formula of the lower band of the magnetic energy:

$$W_m^{(-)} = W_m^{(+)} - W_m'' \quad (17)$$

Two various physical interpretations of the second energy member W_m'' given by (16) may be suggested:

The first of them makes use of a concept of the magnetic vector potential A excited by a fictive electric current that is carried by an edge line $\mathcal{L}^N$ of the N-pole surface. There is to find in [9] the appropriate formula of the magnetic vector potential created by a current-carrying ring co-axial with the cylindrical magnet, in which the same expression as given above in the brackets of (16) occurs. Making necessary substitutions and pointing optionally $z = 0$ at the centre of the N-pole surface the following formula for W_m'' can be rewritten:

$$W_m'' = \frac{V_m \sigma_{ms} A}{2\mu_0 r_m} = \frac{V_m J A}{2\mu_0 r_m} = \frac{V_m}{2r_m} M A \quad (18)$$

where $A = A_\varphi(R_M, -l_M)$ is the φ -component of the magnetic vector potential on the edge line $\mathcal{L}^S$ of the S-pole surface:

$$A = \frac{\mu_0 I_{\text{eq}}}{2k} \left[\left(1 - \frac{k^2}{2} \right) K_0(k) - E_0(k) \right] \quad (19)$$

that is created by the fictive electric current I_{eq} imaginably carried by the edge circle $\mathcal{L}^N$ of the N-pole surface:

$$I_{\text{eq}} = \frac{2l_m \sigma_{mN}}{\mu_0} = \frac{2l_m J}{\mu_0} = 2l_m M \quad (20)$$

The other physical interpretation of W_m'' may be explained by applying of the radial component of the magnetic field strength created by σ_{mN} at the edge line $\mathcal{L}^S$ of the S-pole surface or vice versa. The expression of $H_r = H_r(r_m, 0)$ – with $z = 0$ at a centre of the S-pole surface – may be easily obtained as a negative differential of potential (1) to read:

$$H_r(r_m, 0) = -\frac{\partial \psi_m}{\partial r} = \frac{\sigma_m}{2\mu_0} \int_0^\infty J_1(r_m u) J_0(ru) e^{-|z-l_m|u} du \quad (21)$$

leading for $r = r_m$ to:

$$H_r(r_m, 0) = \frac{\sigma_m}{2\mu_0 k} \left[\left(1 - \frac{k^2}{2} \right) K_0(k) - E_0(k) \right] \quad (22)$$

and resulting in the formula interpreting the second member of W_m as:

$$W_m'' = \frac{V_m l_m}{r_m} J H_r \quad (23)$$

5. EXAMPLE

As an example, let us present in the table 1 the results of calculations that were performed by the use of the specialised test software. Three flat rare-earth Schramberg's magnets of the data given below are considered:

- 1) NdFeB 230/175 w, $B_r = 1,190$ T, $H_{cB} = 890$ kA/m, $r_m = 12,5$ mm, $l_m = 10$ mm, $V_m = 4,19$ cm², catalogue number 11-5210003,
- 2) NdFeB 230/175 w, $B_r = 1,190$ T, $H_{cB} = 890$ kA/m, $r_m = 15,0$ mm, $l_m = 4$ mm, $V_m = 2,83$ cm², catalogue number 11-6019003,
- 3) SmCo5 160/175 h, $B_r = 0,925$ T, $H_{cB} = 710$ kA/m, $r_m = 6,0$ mm, $l_m = 12$ mm, $V_m = 1,36$ cm², catalogue number 11-5120005.

TABLE 1

Magnetic energy created in $\mathfrak{R}^{(3)}$ and in $\mathfrak{R}^{(m)}$ by permanent magnets set in free space.

No.	Magnetic polarization J , T	Magnetic flux density $\bar{B}$, T	Magnetic field strength $\bar{H}$, kA/m	Magnetic energy in $\mathfrak{R}^{(3)}$			Magnetic energy in $\mathfrak{R}^{(m)}$ $\frac{1}{2} V_m \bar{B} \bar{H}$, J
				Upper band $\mu_m^{(+)}$, J	Lower band $\mu_m^{(-)}$, J	Average $\bar{\mu}_m$, J	
1)	1,154	0,594	449	2,600	2,248	2,424	1,328
2)	1,136	0,289	674	0,294	0,252	0,273	0,075
3)	0,919	0,744	139	0,456	0,423	0,439	0,369

The recently elaborated author's test-software package *Ceelin*⁴⁾ was used. The calculation is based under the supposition that the magnets have been previously magnetized in full. After being situated in free space, they are submitted to a natural self-demagnetization that results in the concrete values of magnet quantities that are reckoned by the software. The magnetic polarization $J_z = J$ is considered as homogeneously distributed inside the magnet body, however, the values of both: the magnetic flux density $\bar{B}_z = \bar{B}$ and magnetic field strength $\bar{H}_z = \bar{H}$ are average values on central cross-section of the magnets. The values of the magnetic energy stored in the whole space $\mathfrak{R}^{(3)}$ were calculated by formulae (15) – (17). The value of the magnetic energy stored in the magnet body $\mathfrak{R}^{(m)}$ has been calculated approximately taking into account the average values of both $\bar{B}$ and $\bar{H}$. Exact value differs from $\frac{1}{2} V_m \bar{B} \bar{H}$, indeed; it could be reached by relevant integration over the whole $\mathfrak{R}^{(m)}$ but it is out of main subject of the paper.

The demagnetization effect is commonly evaluated by the demagnetization factor N that takes in the considered case the following values:

- magnet No 1) $\rightarrow N = 0,489$,
- magnet No 2) $\rightarrow N = 0,745$,
- magnet No 3) $\rightarrow N = 0,644$.

⁴⁾ In the *Ceelin* package, the data base contains magnetic curves of 64 modern hard-magnetic materials.

It is evident, that in the shortest magnet (No 2) the demagnetization is most considerable that results in relatively lower value of the magnetic energy.

It is to be seen that, generally, the difference between the upper band and lower band does not exceed some percent, so we may expect that the inaccuracy of the determined value of the magnetic energy $\bar{W}_m$ does not exceed 2,5 %.

6. CONCLUSION

Magnetic feature of a permanent magnet can be modelled by the use of two boundary quantities: *surface monopole density of the magnetic charge*, symbol σ_m , or *magnetizing current sheet*, symbol K_M . It is proved the equivalence of them by revealing that the magnetic energy excited by the magnet and stored in the whole space is the same for both models. The exact result is reached for the case of cylindrical magnet of rather flat shape and not influenced by external fields, under the supposition that the magnet is uniformly magnetized in axial direction and submitted to natural self-demagnetization. Employing in analysis the Lipschitz-Hankel integrals applicable to cylindrical magnets the exact analytical formula for the magnetic energy was achieved (9a), as well as the simple expressions of the upper band (15) and lower band (17) of the expected magnetic energy.

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RÓWNOWAŻNOŚĆ ENERGETYCZNA WIELKOŚCI BRZEGOWYCH NA POWIERZCHNI BIEGUNOWEJ I BOCZNEJ MAGNESU TRWAŁEGO

K. PAWLUK

STRESZCZENIE Wykazano, że w przypadku magnesu trwałego o kształcie cylindrycznym, usytuowanego w wolnej przestrzeni dwie fikcyjne wielkości brzegowe: gęstość powierzchniowa ładunku magnetycznego, którą można przyporządkować powierzchniom biegunowym i okład prądu magnetyzacji wyobrażalny na powierzchni bocznej wytwarzają identyczną energię magnetyczną. Podano dokładne wyrażenia do obliczania energii magnetycznej. Przedstawiono również wyniki obliczeń wykonanych za pomocą specjalnego testowego programu komputerowego opracowanego przez autora.

Prof. Krystyn Pawluk was born in 1926; he received the M.Sc. and El. Eng. degree (1951), and later the Ph.D. degree (1959) from the University of Mining and Metallurgy in Cracow. He became habilitation from the Electrotechnical Institute in Warsaw (1974) and the title of professor in 1979.

In 1951...1961 – assistant-professor at the Department of Electric Machines in the University of Mining and Metallurgy in Cracow, in 1961...1980 – research-engineer at the Department of Electric Machines in the Electrotechnical Institute in Warsaw. 1981...1983 – assistant-professor at the Institut Agronomique et Vétérinaire de l'Université de Mohammed V in Rabat, Morocco, since 1983 – professor at the Department of Fundamental Research in Electrotechnics in the Electrotechnical Institute in Warsaw; at present, he takes a post of substitute of head of the Department.

Since 1985, prof. Pawluk has been nominated an expert of the Polish Committee of Standardisation at the International Electrotechnical Commission and he takes continuously part in works of the IEC committees and working groups: No1 – Terminology, No 25 – Quantities and Units. He is also a president of the national commission of standardisation for terminology and units in electrical engineering. Prof. Pawluk is a member of steering committees of the international conferences:

- International Symposium of Electromagnetic Fields in Electrical Engineering, bi-annual meetings in various countries of Europe,
- International Seminar on Fundamentals of Electrotechnics and Circuit Theory, annual meetings in Poland.

Prof. Pawluk develops large research activity: project leaderships, heading of laboratories, he is the author of numerous scientific works (about 150 bibliographic items), conference reports, reviewing of scientific works, opponent of doctor thesis, etc. His main research field is the following:

- electric machines, in particular synchronous machines: measurement of transient quantities, dynamic behaviour and design of turbo-generators and motors,
- electric and magnetic fields in objects of electrical technology; boundary element method, inverse problems, stationary and quasi-stationary fields,
- terminology and units in electrical technology.



